International Space Station Centrifuge Rotor Models A Comparison of the Euler-Lagrange and the Bond Graph Modeling Approach

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Abstract

The assembly and operation of the International Space Station (ISS) require extensive testing and engineering analysis to verify that the Space Station system of systems would work together without any adverse interactions. Since the dynamic behavior of an entire Space Station cannot be tested on earth, math models of the Space Station structures and mechanical systems have to be built and integrated in computer simulations and analysis tools to analyze and predict what will happen in space. The ISS Centrifuge Rotor (CR) is one of many mechanical systems that need to be modeled and analyzed to verify the ISS integrated system performance on-orbit. This study investigates using Bond Graph modeling techniques as quick and simplified ways to generate models of the ISS Centrifuge Rotor. This paper outlines the steps used to generate simple and more complex models of the CR using Bond Graph Computer Aided Modeling Program with Graphical Input (CAMP-G). Comparisons of the Bond Graph CR models with those derived from Euler-Lagrange equations in MATLAB and those developed using multibody dynamic simulation at the National Aeronautics and Space Administration (NASA) Johnson Space Center (JSC) are presented to demonstrate the usefulness of the Bond Graph modeling approach for aeronautics and space applications.

1. INTRODUCTION

The International Space Station (ISS) is a complex space vehicle, which will house many space-based science

experimental systems. One of these experiments is the ISS Centrifuge Rotor (CR) system, which allows biological experiments at 0-2 gravitational accelerations. The CR is housed in the Centrifuge Accommodation Module (CAM), and it is being developed and built by the Japanese Aerospace Exploration Agency (JAXA) for the ISS program. The ISS CR consists of a 2.5 m diameter rotor housing containing up to four biological habitats, an automatic balancing system (ABS), a stator with a spin motor, and a vibration isolation mechanism (VIM) between the stator and the CAM housing or shroud. As the CR rotates, imbalances in the rotor due to habitat configuration and/or rodent motion can cause vibrations that impact other micro gravity experiments aboard the Space Station and/or other ISS system performance (e.g. flight control system). The vibration isolation mechanism serves the dual purpose of damping the CR vibrations and providing measured stator motion for the centrifuge automatic balancing system. The ABS corrects for the rotor imbalances by positioning balance masses within the rotor to minimize the stator motion. The physical interactions between the CR and the ISS through the VIM can impact both the CR and ISS systems performance and stability (e.g. ABS and flight control system). Prior to the CR operation, integrated tests and verification analyses must be performed to ensure safe and successful operations on-orbit. Therefore, analysis models of the CR system are needed to understand and verify the CR system design and its interactions with the ISS.

The analysis of the CR system requires both simple and complex, and linear and non-linear models. Simple, linear models of the CR system are useful for control system

design and development purposes. Complex and detailed multi-body non-linear models are needed for design and integrated verification analyses. Various computational methods and computer programs are available for modeling of multi-body systems consisting rigid and/or flexible bodies with inertias, springs, dampers, and servomotors/ actuators, etc... [1,2]. This paper investigates the use of the Bond Graph technology and the integrated Computer Aided Modeling Program with Graphical input (CAMP-G) -MATLAB/Simulink software package to develop analysis and simulation models of the ISS CR. Two Bond Graph/CAMP-G modeling techniques are presented. An overview of the comparison of the CR equations of motion derived using the Bond Graph/CAMP-G method and those derived using Lagrange/Newton-Euler approach is presented. The verification of the Bond Graph CR model using time-domain simulation results is also presented.

2. THE SPACE STATION CENTRIFUGE ROTOR SYSTEM

The ISS CR system consists of three main components: the rotor, the stator, and the vibration isolation mechanism (Figure 1). The rotor contains the habitats and its support systems, and static and dynamic balance masses for the automatic balance system (ABS). The stator contains the spin motor. The vibration isolation mechanism (VIM) consists of joined bodies/links with springs, passive dampers and active dampers. The VIM is connected to the base or shroud and the ISS. The VIM constrains the stator only in the rotational degree of freedom of the CR, and restricts the other five degrees of freedom to balance and isolate the CR motion from the Space Station, and vice versa.

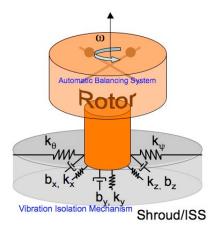


Figure 1. Simple Representation of the ISS Centrifuge Rotor

2.1. Five Degrees of Freedom Model

The five degrees of freedom (5-DOF) model assumes that the stator and the rotor is one body, and ignores the

rotational effects of the rotor. Based on the coordinates system in Figure 2 the stator/rotor can translate in the x, y, and z direction and can tilt (small rotation) about the y and z-axis.

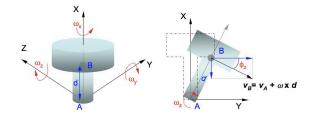


Figure 2. Kinematic Analysis of the Velocity of the Center of Mass of the Stator/Rotor

2.1.1. Kinematic Equations

In order to model this system, the kinematics of the system is determined. In doing so, the velocity components of the center of mass B are critical to determine the geometry and functionality of the physical model. The connectivity of the elements of the system is used to establish the constraints and forces between the different rigid bodies. Here, the translation motion in x, y, and z is measured with respect to an inertial frame of reference fixed on the stator of the system. The system has been analyzed for small oscillations about the y and z-axis, around the vertical (zero degrees) position. The velocity of point B is the velocity of point A plus the relative velocity vector between the two. This involves the cross product of the angular velocity vector and the position vector in order to calculate the tangential velocity of point B. Shown in Figure 2 is the decomposition of the velocity vectors. It shows also how each term contributes to the calculation of the vector velocity of B. Consequently; the kinematic equations of the velocity of the center of mass of point B are as follow, taking into consideration the rotational motions.

$$\begin{aligned} V_{Bx} &= V_{Ax} - \omega_z \cdot d \cdot \sin \phi_z \cos \phi_y - \omega_y \cdot d \cdot \sin \phi_y \\ V_{By} &= V_{Ay} + \omega_z \cdot d \cdot \cos \phi_z \cos \phi_y + \omega_x \cdot d \cdot \sin \phi_y \\ V_{Bz} &= V_{Az} - \omega_y \cdot d \cdot \cos \phi_z \cos \phi_y + \omega_x \cdot d \cdot \sin \phi_z \cos \phi_y \end{aligned} \tag{1}$$

2.1.2. Bond Graph Model

Using CAMP-G, the following Bond Graph structure is generated. Considering the following sign convention, positive for half arrow pointing into the 0 junction and negative for half arrow pointing away from the 0 junction. Using such convention, the kinematic equations (1) are represented by the Bond Graph structure shown in Figure 3. The 0 junctions have been used to show the sums and subtractions of the velocity terms shown in the kinematic equations. The modulated transformer (MTF) elements are defined to represent the Sine and Cosine geometric and

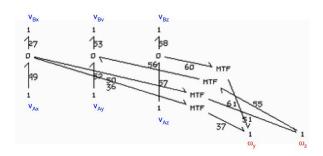


Figure 3. Bond Graph Structure Representing Kinematic Velocity Equations

kinematic transformations. To complete the Bond Graph model, physical elements, springs, dampers, masses, and external inputs are added. For the 5-DOF CR model, there are three translational degrees of freedom along the x, y, and z-axis and two rotational degrees of freedom about the y and the z-axis, as shown in Figure 4. The VIM springs and dampers are added in each direction. Each translational degree of freedom has one spring and two dampers (one passive and one active) elements. Each rotational degree of freedom has only one spring element. The CAMP-G software is used to assemble this Bond Graph model in graphical form.

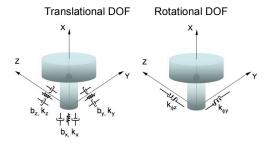


Figure 4. VIM Springs and Dampers

Using the menu of available Bond Graph elements in CAMP-G, the 5-DOF CR model is constructed as shown in Figure 5. The inertia elements are added to the 1's junctions, which represent the summation of forces in the x, y, and z directions. These are represented by the mass/inertia elements on the bonds I34, I42, and I48. For this modeling approach, the mass of the VIM linkages is lumped with that of the stator/rotor for the corresponding motions in x, y, and z directions. Spring and dampers for the VIM are added in each corresponding direction. The C elements, representing the springs, are attached to the 1 junction along with the R elements, representing the dampers. The C's and R elements are in between the stator/rotor and the shroud, which is rigidly attached to the Space Station. This model allows the possibility of accommodating inputs from the shroud/ISS to study the time response and frequency response to any output in the CR system in relation to the inputs from the

ISS in the directions represented by the SF elements. This is possible because symbolic and numeric expression for any energy variables (i.e. momentum and displacement) are available in the CAMP-G generated model.

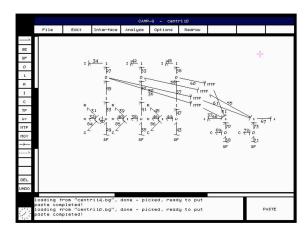


Figure 5. CAMP-G Bond Graph Model of CR

2.1.3. Forcing Function due to Rotor Imbalance

The Bond Graph model can also accommodate input forces directly to the rotor/stator (i.e. to the I elements). One purpose for this is to apply forces to the center of mass of the stator/rotor, or at some other point on the stator/rotor, to emulate centrifugal forces from an unbalanced spinning rotor. The complete 5-DOF Bond Graph model with up to ten inputs is shown in Figure 6. The inputs from the Space Station in terms of velocity are represented by SF29, SF35, and SF43 in the x, y, and z direction, respectively, and by SF68 and SF71 for the rotations about y and z-axis. Five

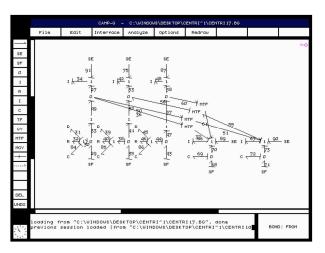


Figure 6. Complete 5 Degrees of Freedom Bond Graph Model of CR

external force and torque inputs to the stator/rotor are represented by SE91, SE75, SE87, SE89, and SE90. With

these interfaces, rotor motions due to rotation and imbalance can be studied in this model by applying the appropriate SE forces and torques as functions of the rotor rotational speed.

2.2. Eleven Degrees of Freedom Model

The 5-DOF model has assumed a single body for the stator/rotor, and externally calculated forcing function applied to the stator/rotor to simulate the rotational dynamics. To model the three dimensional rotor dynamics in Bond Graph, first, Newton-Euler equations a three-dimensional rigid body motion is examined. Similar to the exercise in part A1, the Newton-Euler equations for the linear and angular momentum [3] are used to construct the Bond Graph structure for translational and rotational motion of the rotor (Figure 7). The combination of the translation and rotation Bond Graph structures can be used to create a three dimensional dynamic model of the rotor.

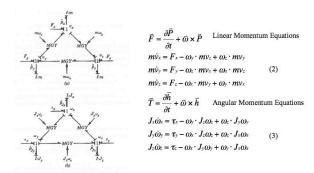


Figure 7. Bond Graph for 3-D Rigid Body Motion

As developed in Section 2.1, the stator is in the original inertial coordinate frame as shown in Figure 2. The three dimensional dynamic equations for the rotor are based on the body fixed coordinate frame. Figure 8 shows the rotor in a different coordinate frame in relation to the inertial coordinate frame used for the stator. To establish the interactions between the stator and the rotor a Cardan angle coordinate transformation is necessary. The multiport MTF element is used for this purpose, and is placed between the

developed model for the stator and the body fixed model for the rotor. This way the inputs can be directly entered in relation to the body fixed axis, which rotates with the rotor, and the effects studied in the inertial frame.

The three dimensional dynamic model of the rotor, which has six

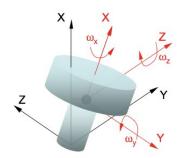


Figure 8. Introduction of CR Rotating Frame

degrees of freedom, is combined with the five degrees of freedom model of the stator and VIM to make the eleven degrees of freedom model. This model is assembled in CAMP-G, and is as shown in Figure 9. Several characteristics of this model are worth noting. The development of the equations is made with principal axis attached to the center of mass of the rotor, thus avoiding many terms that relate to the products of inertia. The model has time variant coefficients; particularly all modulated gyrators (MGY) are time dependent by the very nature of Euler's equations, which couple the gyroscopic effects of three-dimensional dynamics.

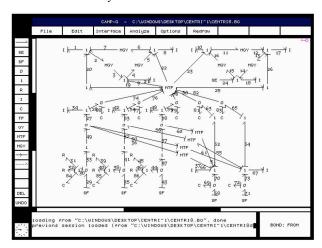


Figure 9. Bond Graph 11 Degrees of Freedom CR

3. VERIFICATION OF BOND GRAPH MODEL

As discussed in Section 2, the Bond Graph/CAMP-G approach can be used to generate simple and possibly more complex model of the ISS CR. The verification of the Bond Graph model is done in two steps. In step 1, the eigenvalues and natural frequencies of the two models are compared, for the translational degree of freedom only and for both translational and rotational DOF. In step 2, time domain results from the two simulation models are compared. To assess and verify the Bond Graph model of the CR, an 'equivalent' 5-DOF CR model, derived using Lagrange/Newton-Euler approach, is compared with the CAMP-G computer-generated model.

3.1. Lagrange/Newton-Euler 5 DOF Model

The Lagrange/Newton-Euler 5 DOF (CR-5DOF) model is a four-body model of the CR system with 5 degrees of freedom. Similar to the assumptions used in the Bond Graph model, the rotor/stator is assumed to be one body, and the three joined bodies with springs and dampers making up the VIM (Figure 10). The linear equations of motion for this model are written as (see Appendix)

$$M\ddot{q} + (G+C)\dot{q} + Kq = F$$
, (4)

where M is the mass matrix, G is the skew-symmetric gyroscopic matrix which is linearized about a constant rotor spin rate (ω) , C is the damping matrix, K is the stiffness matrix, Q is the vector containing the degrees of freedom, and P is the load vector. The degrees of freedom of this system are as defined for the CAMP-G model,

$$q = \begin{cases} x \\ y \\ z \\ \phi_y \\ \phi_z \end{cases}$$

The above second order equations are linearized into standard state space form in MATLAB for analysis and simulation.

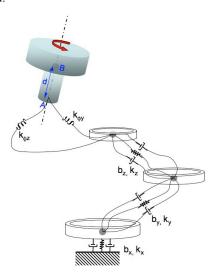


Figure 10. CR -5DOF Model

3.2. Eigenvalues and Frequencies Comparison [4]

Since the CAMP-G software generates the equations of motion symbolically, the physical parameters are easily identified, and initialized with the appropriate mass/inertia, stiffness, and damping values via the generated CAMPGMOD.M module. With the CAMPGNUM module, containing the numeric state space matrices, transfer functions and characteristic equation, eigenvalues and natural frequencies of the system are calculated for comparison with those calculated from the CR-5DOF model with a zero spin rate, $\omega=0$.

3.2.1. 6th Order State Space System

Initially, the angular (tilting) motions of the stator/rotor are excluded in the CR models to assess only a 6th order state space system (i.e. $q^T = \{x \mid y \mid z \mid \dot{x} \mid \dot{y} \mid \dot{z}\}$). The

eigenvalues and frequencies for this system compared quite well between the two models, Lagrange/Newton-Euler approach and Bond Graph/CAMP-G approach, as shown in Table 1.

3.2.2. 10th Order State Space System

Table 1. Eigenvalues with Locked Rotational DOF

#	Lagrange/Newton-Euler Approach			Bond Graph Approach		
	Eigenvalues	ω rad/s	ζ Damping	Eigenvalues	ω rad/s	ζ Damping
1	-0.3960 ± 0.7391i	0.8385	0.4722	-0.3960 ± 0.7391i	0.8385	0.4723
2	-0.2046 ± 0.8273i	0.8522	0.2401	$\text{-}0.2047 \pm 0.8273 i$	0.8522	0.2401
3	-0.2089 ± 0.8353i	0.8611	0.2426	$-0.2089 \pm 0.8354i$	0.8611	0.2426

Table 2. Eigenvalues with All 5-DOF (10th Order State Space)

	Lagrange/New	ton-Euler Ap	proach	Bond Graph Approach		
#	Eigenvalues	ω rad/s	ζ Damping	Eigenvalues	ω rad/s	ζ Damping
1	$-0.186 \pm 0.814i$	0.835	0.222	$-0.184 \pm 0.813i$	0.833	0.221
2	$-0.396 \pm 0.739i$	0.838	0.472	$-0.396 \pm 0.739i$	0.838	0.472
3	$-0.189 \pm 0.822i$	0.843	0.224	-0.188 ± 0.821i	0.842	0.223
4	-0.164 ± 3.250i	3.250	0.050	-0.188 ± 3.320i	3.320	0.057
5	-0.174 ± 3.280i	3.280	0.053	-0.188 ± 3.320i	3.320	0.057

Similarly, the complete 5-DOF models (10th order state space) are compared and the eigenvalues and frequencies for that system also compared quite well, as shown in Table 2. With the rotational degrees of freedom, however, noticeable difference can be seen. The small difference between the two results is due to the difference in the mass matrix. The CAMP-G 5-DOF model mass matrix is diagonal, while the CR-5DOF model has coupling terms between the translational and rotational degrees of freedom.

3.2.3. Time Domain Simulation Comparison

Time domain simulation of these CR models uses an externally applied forcing function to represent the disturbance from mass imbalance. The forcing functions due to rotor imbalance can be expressed in the rotating frame (Figure 11) as

$$f_{\xi} = 0$$

$$f_{\eta} = M\varepsilon(\omega^{2}\cos\beta + \dot{\omega}\sin\beta)$$

$$f_{\xi} = M\varepsilon(\omega^{2}\sin\beta - \dot{\omega}\cos\beta)$$

$$f_{\phi\xi} = -J_{xx}\dot{\omega}$$

$$f_{\phi\eta} = -J_{xy}\dot{\omega} + (I_{p} - I_{t})\alpha\omega^{2}\cos\gamma$$

$$f_{\phi\xi} = -J_{xz}\dot{\omega} + (I_{p} - I_{t})\alpha\omega^{2}\sin\gamma$$
(5)

where M, I_p , and I_t are, respectively, the mass, polar moment of inertia and transverse moment of inertia of the rotor, ω is the rotor spin rate, and $\dot{\omega}$ is the spin acceleration. The inertia values are with respect to the center of mass of the rotor. The geometric parameters to characterize the rotor imbalance (i.e. ε , β , J_{xx} , J_{xy} , J_{xz} , α , and γ) can be determined from the masses and its locations in the rotor coordinates system. The forcing functions are calculated for a loading condition where an imbalanced rotor is constantly accelerated to a spinning rate of 0.7 revolution per second, and applied to the effort sources in the 5-DOF CAMP-G simulation model (i.e. SE91, SE95, SE87, SE89, and SE90).

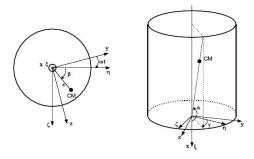


Figure 11. Phase Angles and Rotor Rotating Coordinate System

Initially, comparison of the CR simulation results reveals another subtle difference between the CAMP-G 5-DOF model and the Lagrange/Newton-Euler CR-5DOF model. The difference is attributed to the gyroscopic

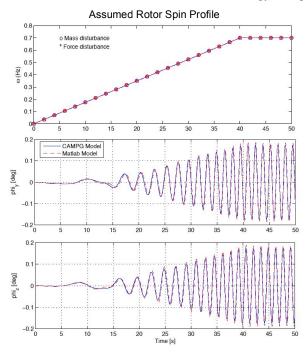


Figure 12. Stator/Rotor Rotational Motion Comparison

coupling terms, which are not modeled in the CAMP-G simulation. In the Lagrange/Newton-Euler CR-5DOF model, the gyroscopic matrix, G, includes two nonzero elements containing $I_x\omega$. Since the CAMP-G 5-DOF model does not include these coupling terms due to the spinning rotor, the simulation results do not match. However, this issue can easily be remedied by including the effects of these terms in the SE elements (i.e. on the forcing function side).

$$SE89 = SE89 + I_x\omega(P66/I66)$$

 $SE90 = SE90 - I_x\omega(P67/I67)$

Once the gyroscopic coupling terms are included in the CAMP-G simulation the time domain results match quite well between the two models. Figures 12 and 13 show, respectively, the rotational and translational motion of the stator/rotor for the assumed spin profile.

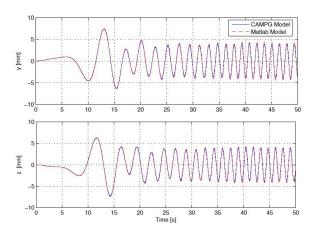


Figure 13. Stator/Rotor Translational Motion Comparison

4. CONCLUSION

As demonstrated in this paper, the Bond Graph/CAMP-G modeling approach was successfully used to generate two simple models of the ISS Centrifuge Rotor system. Two Bond Graph models have been presented in detail, but other possibilities with more complex modeling can be considered. The 5-DOF CR model generated by the CAMP-G process was analyzed and compared against similar model derived using traditional Lagrange/Newton-Euler method. The analysis and verification of the 5-DOF CR model, both in frequency and time domain, showed that the CAMP-G process could quickly and accurately generate analysis and simulation model of the CR system. The CAMP-G generated model is very useful for design and analysis of aerospace systems such as the ISS centrifuge rotor. The CAMP-G generated state space matrices contain symbols representing physical parameters, which are replaced with numerical values once the physical parameters are initialized. Therefore, even if the system is linear in the state space form, non-linear and time variant analyses can be done by providing additional calculations to supply numerical values for varying coefficients.

Although in this exercise, the traditional Lagrange/ Newton-Euler model was used to verify the CAMP-G model, the results demonstrated that the Bond Graph model process in CAMP-G could be used to quickly generate model and simulation for analysis and verification of other models developed by other means.

Appendix

This appendix elaborates on the equations of motion for the **CR-5DOF** model and the matrix elements contained in the equations. The CR-5DOF model is a four-body model of the centrifuge rotor system (Figure 10) with 5 DOFs,

$$M\ddot{q} + (G+C)\dot{q} + Kq = F$$
 ,where
$$q = \begin{cases} x \\ y \\ z \\ \phi y \end{cases}$$
 (A1)

The mass matrix, M, is defined as follow,

$$M = \begin{bmatrix} M_R + M_1 + M_2 + M_3 & 0 & 0 & 0 & 0 \\ 0 & M_R + M_1 & 0 & 0 & -M_R d \\ 0 & 0 & M_R + M_1 + M_2 & M_R d & 0 \\ 0 & 0 & M_R d & I_d + M_R d^2 & 0 \\ 0 & -M_R d & 0 & 0 & I_d + M_R d^2 \end{bmatrix}$$
(A2)

where I_d is the rotor average moment of inertia about the y-axis and z-axis. The gyroscopic matrix, G, is a skew-symmetric matrix with two non-zero elements (angular momentum about the rotor spin axis).

The damping and stiffness matrices, D and K, are both diagonal matrices containing the equivalent damping and stiffness between each DOF.

$$C = diag(\{b_{tx}, b_{ty}, b_{tz}, b_{ry}, b_{rz}\})$$
 (A4)

$$K = diag(\{k_{tx}, k_{ty}, k_{tz}, k_{ty}, k_{rz}\})$$

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Biography

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